

# Liquid-gas phase transition in a two-components isospin lattice gas model for asymmetric nuclear matter

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## Abstract

A two-components isospin lattice gas model has been employed to study the liquid-gas phase transition for asymmetric nuclear matter. An additional degree of freedom, namely, the asymmetry parameter  $\alpha$  has been considered carefully for studying the phase transition. We have shown that under the mean field approximation, the liquid-gas phase transition given by this model is of second order. The entropy continues at the phase transition point. The binodal surface is addressed.

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An one-component lattice gas model with one type of atom has been found to be successful for describing the liquid-gas phase transition (LGPT) [1]. An isospin lattice gas model which assumes a lattice with each site either being vacant or occupied by proton and neutron has been employed by many authors to study the LGPT of *symmetric* nuclear matter recently [2-6]. This model is mapped into a spin-1 Ising model. It has been proven that the LGPT of this model is of first order under the Bragg-Williams mean field approximation. The chemical potential continues at the phase transition point but its first order derivatives, namely, entropy and volume, are discontinuous. This result is reasonable if we notice that the number of protons  $N_+$  equals to the number of neutrons  $N_-$  for symmetric nuclear matter and the symmetric isospin lattice gas model is, in fact, a system with one-component.

However, as was pointed out by ref.[7-12], the LGPT for a multi-components system is of second order, i.e. the entropy continues and the second order derivatives of chemical potential are discontinuous. This is because the degrees of freedom increase according to the Gibbs phase rule, and then the binodal surface has greater dimensionality. Since the space occupied by nuclear matter addressed by ref.[7-12] is continuous, the above conclusion is of course limited to a continuous manifold of space structure. It is of interest to extend this study to a two-components system with lattice space structure.

The isospin lattice gas model for asymmetric nuclear matter suggested by ref.[13] is a typical two-components model because  $N_+ \neq N_-$ , the number of protons and neutrons are independent. We will employ this model to reconsider this problem. In conflict with ref.[13], we will show the LGPT is of second order for two-components isospin lattice gas model and the binodal surface will end at two points with  $\alpha = 0$  and  $\alpha = 1$  where  $\alpha$  is the asymmetric parameter

$$\alpha = \frac{N_- - N_+}{N_- + N_+} \quad (1)$$

At these two end-points the LGPT becomes first order because the nuclear matter reduces to symmetric and neutron matter, and the lattice gas model reduces to one-component.

The Hamiltonian of lattice gas model is[13]

$$H = - \sum_{\langle ij \rangle} J_{ij} \tau_{zi} \tau_{zj} - h \sum_i \tau_{zi} \quad (2)$$

where  $\langle ij \rangle$  denotes a nearest neighbor pair,  $h$  is a constant and the interaction strength parameter  $J_{ij}$  satisfy

$$J_{ij} = \begin{cases} J_{ij} & \text{for nearest neighbor distance} = a \\ \infty & \text{for neighbor distance} = 0 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

The isospin  $\tau_z$  is

$$\tau_z = \begin{cases} -1 & \text{for neutron} \\ 0 & \text{for vacancy} \\ +1 & \text{for proton} \end{cases} \quad (4)$$

The Hamiltonian can be written as,

$$H = -J_s(N_{++} + N_{--}) + J_d N_{+-} - h(N_+ - N_-) \quad (5)$$

where  $N_{++}$ ,  $N_{--}$  and  $N_{+-}$  represent the nearest neighbor pairs of proton-proton, neutron-neutron and proton-neutron respectively, and the interaction strength parameter  $J_s$  for  $pp$  and  $nn$  pairs, and  $J_d$  for  $pn$  pairs. Introducing

$$r = \frac{N_0}{N}, s = \frac{N_+ - N_-}{N}, N = N_+ + N_- + N_0 \quad (6)$$

where  $r$  denotes the relative emptiness and  $s$  plays the similar role as that of asymmetric parameter  $\alpha$ .  $s$  is not a new parameter. We introduce it for comparing with ref.[13] only. By using the Bragg-Williams mean field approximation [14]

$$\frac{N_{++}}{\frac{1}{2}\gamma N} \approx \left(\frac{N_+}{N}\right)^2, \frac{N_{--}}{\frac{1}{2}\gamma N} \approx \left(\frac{N_-}{N}\right)^2, \frac{N_{00}}{\frac{1}{2}\gamma N} \approx \left(\frac{N_0}{N}\right)^2 \quad (7)$$

where  $\gamma$  denotes the number of nearest neighbors of any given site, and  $\frac{1}{2}\gamma N$  is the total number of pairs. We can rewrite Eq.(5) as

$$H(r, s, N) = -C_1 N s^2 - C_2 N (1 - r)^2 - h N s \quad (8)$$

where

$$C_1 = \frac{(J_s + J_d)\gamma}{4}, C_2 = \frac{(J_s - J_d)\gamma}{4} \quad (9)$$

The grand partition function of our system is

$$Q_G = \sum_{r,s} g(r, s, N) z_+^{N_+} z_-^{N_-} e^{-\beta H} = \sum_{r,s} g(r, s, N) z_+^{\frac{N}{2}(1-r+s)} z_-^{\frac{N}{2}(1-r-s)} e^{-\beta H} \quad (10)$$

where  $\beta = \frac{1}{k_B T}$ , the factor

$$g(r, s, N) = \frac{N!}{N_0! N_+! N_-!} = \frac{N!}{(Nr)! \left[\frac{N}{2}(1-r+s)\right]! \left[\frac{N}{2}(1-r-s)\right]!} \quad (11)$$

the fugacity  $z_+ = e^{\beta\mu_+}$  and  $z_- = e^{\beta\mu_-}$  for proton and neutron respectively. For two-components system, in general, the chemical potential of proton  $\mu_+$  does not equal to the chemical potential of neutron  $\mu_-$  because  $N_+ \neq N_-$ . According to the Gibbs phase rule, the degrees of freedom of a system is

$$f = k - \varphi + 2 \quad (12)$$

where  $k$  and  $\varphi$  are the number of components and phases respectively. A two-components system has one more degree of freedom than that of one-component. We choose the asymmetric parameter  $\alpha$  (or  $s$ ) as the additional degree of freedom for two-components lattice gas model as that of nuclear matter models with continuous space structure [7-12]. Therefore, we have two independent parameters, namely  $r$  (corresponding to the density) and  $s$  (corresponding to the asymmetry of protons and neutrons) in a two-components lattice gas

model for asymmetric nuclear matter. This is the basic difference between our calculation and that of ref.[13].

In the thermodynamic limit, the sum in Eq.(10) can be replaced by its most dominant term [3,13,14]. Since we have two independent parameters  $r$  and  $s$ , the extreme values  $(\bar{r}, \bar{s})$  satisfy

$$\frac{\partial \ln Q_G}{\partial r} \Big|_{r=\bar{r}, s=\bar{s}} = 0 \quad (13)$$

and

$$\frac{\partial \ln Q_G}{\partial s} \Big|_{r=\bar{r}, s=\bar{s}} = 0 \quad (14)$$

We must use two equations to determine the extreme values of  $r$  and  $s$ . Using Stirling's formula and Eqs.(13) (14), we obtain

$$\beta\mu_+ = \log z_+ = -\log \bar{r} + \log \frac{1 - \bar{r} + \bar{s}}{2} - 2\beta C_2 (1 - \bar{r}) - 2\beta C_1 \bar{s} - \beta h \quad (15)$$

$$\beta\mu_- = \log z_- = -\log \bar{r} + \log \frac{1 - \bar{r} - \bar{s}}{2} - 2\beta C_2 (1 - \bar{r}) + 2\beta C_1 \bar{s} + \beta h \quad (16)$$

Only the maximum of the extreme values gives us the most dominant contribution of Eq.(10) and corresponds to stable state. Taking  $h = 0$  to neglect the effect of the "external field", we find Eq.(15) reduces to Eq.(16) and  $\mu_+ = \mu_-$  when  $\bar{s} = 0$ , the asymmetric nuclear matter becomes symmetric one and two-components becomes one-component. The symmetric nuclear matter is the specific case of our theory.

By means of the grand partition function, we can obtain other thermodynamical quantities such as pressure, baryon density and entropy per baryon easily. They are

$$\begin{aligned} P_{gas}(\bar{r}, \bar{s}, \beta) = & C_1 \bar{s}^2 + C_2 (1 - \bar{r})^2 - \frac{(1 - \bar{r} + \bar{s})}{2\beta} \log \frac{(1 - \bar{r} + \bar{s})}{2} - \frac{(1 - \bar{r} - \bar{s})}{2\beta} \log \frac{(1 - \bar{r} - \bar{s})}{2} \\ & - \frac{\bar{r}}{\beta} \log \bar{r} + h\bar{s} + \frac{(1 - \bar{r} + \bar{s})}{2\beta} \log z_+ + \frac{(1 - \bar{r} - \bar{s})}{2\beta} \log z_- \end{aligned} \quad (17)$$

$$\rho = 1 - \bar{r} \quad (18)$$

$$\begin{aligned} \frac{S}{B} = & \frac{S}{N_+ + N_-} = \frac{1}{1 - \bar{r}} \frac{S}{N} \\ = & -\frac{1}{1 - \bar{r}} \left( \frac{(1 - \bar{r} + \bar{s})}{2} \log \frac{(1 - \bar{r} + \bar{s})}{2} + \frac{(1 - \bar{r} - \bar{s})}{2} \log \frac{(1 - \bar{r} - \bar{s})}{2} + \bar{r} \log \bar{r} \right) \end{aligned} \quad (19)$$

Now we are in the position to discuss the LGPT of our model. The two-phase coexistence equations read

$$\mu_+^L = \mu_+^V \quad (20)$$

$$p^L = p^V \quad (21)$$

$$\mu_-^L = \mu_-^V \quad (22)$$

where subscripts of one phase  $L$  and  $V$  stand for liquid and gas respectively. The parameters  $C_1$ ,  $C_2$  are determined as follows: to have an attractive nearest neighbor interaction, we have  $J_s > 0$  and  $J_d < 0$ . In this case  $C_2$  is positive. We choose the values of  $C_1$  and  $C_2$  to reproduce the binding energy ( $-16MeV$ ) for symmetric nuclear matter at saturation point when  $T = 0K$ , and find

$$C_1 = 3.2MeV, \quad C_2 = 16.0MeV$$

Our numerical results are shown in figures(1-4). In Fig.1, we show the isotherms for fixed  $\alpha = 0.50$  but different temperature  $T = 6.00, 6.90, 7.95, 8.58$  and  $8.70MeV$  respectively. The stable states of isotherms corresponding to the most dominant term in the summation of Eq.(10) is drawn by the solid curves. The dotted curve which encloses the transition region represents the slice of binodal surface with constant asymmetric parameter. The critical temperature for  $\alpha = 0.50$  asymmetric nuclear matter  $T_c = 8.58MeV$ .

The chemical isobar of proton and neutron vs.  $\alpha$  curves at fixed temperature  $T = 7.50MeV$  and  $p = 1.12MeV fm^{-3}$  are shown in Fig.2. In this figure, all metastable and unstable states are neglected. The Gibbs conditions Eqs.(20-22) for phase transition demand equal pressures and equal chemical potentials for two phase with different  $\alpha$ . The collection of all such  $\alpha_1(T, p)$  and  $\alpha_2(T, p)$  form the binodal surface. The two desired solutions form the edges of a rectangle and can be found by using the geometrical construction shown in Fig.2. We find the end points of the phase boundary are just mapped at the edges of the rectangle. This situation is the same as that of the models with continuous space manifold [7-12].

The section of binodal surface at  $T = 7.50MeV$  are shown in Fig.3. We find that the end-points of binodal surface are fixed at  $\alpha = 0$  and  $\alpha = 1$  respectively. The pressure and chemical potential unchange during phase transition. This is just the behavior of one-component system. For cases  $0 < \alpha < 1$ , the physical behavior of Fig.3 can be explained as follows: assume that the system is initially prepared with  $\alpha = 0.5$  (gas phase), during isothermal compression, the two phase coexistence region is encountered at point  $A$ , and the liquid phase emerges at point  $B$ . The isothermal compression process continues to the point  $C$  of liquid-gas coexistence region, the density  $\rho_+$  and  $\rho_-$  for protons and neutrons satisfy

$$\begin{aligned} \rho_+ &= \lambda \rho_+^V + (1 - \lambda) \rho_+^L \\ \rho_- &= \lambda \rho_-^V + (1 - \lambda) \rho_-^L \quad (0 < \lambda < 1) \end{aligned} \quad (23)$$

where  $\lambda$  refers to the volume fraction of gas phase, when the process evolves to the point  $F$ , the gas phase disappears and the system becomes liquid.

Finally, we investigate the entropy and the order of LGPT. The entropy per baryon as a function of pressure for different asymmetric parameter  $\alpha$  are shown in Fig.4. The states denoted by dotted curves in the intermedial states during the phase transition is evaluated according to

$$\frac{S}{B} = \lambda \frac{S^V \rho^V}{B^V \rho} + (1 - \lambda) \frac{S^L \rho^L}{B^L \rho} \quad (24)$$

where  $\rho = \rho_+ + \rho_-$ ,  $\rho^V = \rho_+^V + \rho_-^V$ ,  $\rho^L = \rho_+^L + \rho_-^L$  and the conservation law Eqs.(23). The curves in Fig.4 show the order of phase transition transparently. For symmetric nuclear matter ( $\alpha = 0$ ) and for neutron matter ( $\alpha = 1$ ), the entropy discontinues at the phase-transitional point. The LGPT is of first order for one-component system. For asymmetric nuclear matter,  $0 < \alpha < 1$ , the entropy continues at the phase-transitional point and the LGPT is of second order.

In summary, contrary to ref.[13], an additional degree of freedom  $\alpha$  which makes the chemical potential of proton does not equal to that of neutron has been considered for asymmetric isospin lattice gas model. We have shown that under Bragg-Williams mean field approximation, the LGPT in a two-components isospin lattice gas model for asymmetric nuclear matter is of second order. At the phase transition point, the entropy continues. The binodal surface. The binodal surface for fixed temperature  $T = 7.50 MeV$  has been given.

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## FIGURE CAPTIONS

**Fig.1** The isotherms for fixed  $\alpha = 0.50$  and different temperatures  $T = 6.00, 6.90, 7.95, 8.58$  and  $8.70 MeV$  respectively.

**Fig.2** Chemical potentials for proton and neutron as a function of  $\alpha$  at  $p = 1.12 MeV$  and  $T = 7.50 MeV$ .

**Fig.3** The section of binodal surface for  $T = 7.50 MeV$ .

**Fig.4** Entropy per baryon vs. pressure for asymmetric parameter  $\alpha = 0.0, \alpha = 0.6, \alpha = 0.8$  and  $\alpha = 1.0$  at  $T = 7.5 MeV$ .